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LETTER TO THE EDITOR

Emission properties of electrons in two-level systems driven by DC–AC fields

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Abstract. The emission spectrum of an electron in a two-level system driven by DC–AC fields has been studied. We show that the emission spectrum generally consists of a static component, at low frequency, and doublets at all harmonics of the laser frequency. The phenomenon of low-frequency generation (LFG) is observed, and the conditions under which the LFG is intense and the localization is strong are obtained. The signature of the localization condition for this system in the emission spectrum is manifested.

The behaviour of charged particles in a symmetric double well or a biased heterostructure double well driven by a strong laser has been studied in great detail in several articles [1–5]. A remarkable feature, called the coherent destruction of tunnelling, was discovered in such systems by Grossmann *et al* [1]. They found that an electron initially localized in one of the wells will remain in that well throughout the whole driving process if the laser power and frequency are chosen appropriately. Since the double-well configuration can be fairly well reproduced by a two-level Hamiltonian [6], there is considerable interest in recent studies for such systems [7, 8]. Very recently, the present authors investigated the evolution problem of a two-level system under the influence of a DC–AC field [9], where a fan structure in the parameter space for generating the dynamic localization was found. In this article, we study the emission properties of this system, to find the signature of the localization condition, i.e., the fan structure, in the emission spectrum.

The Hamiltonian that we consider here can be approximately written as

$$H = \hbar \Delta \sigma_x + V(t) \sigma_z . \tag{1}$$

Here Δ is the splitting parameter, $V(t)$ is a driving force

$$V(t) = \mu E_0 + \mu E \cos \omega t \tag{2}$$

where μ is the transition dipole connecting two levels, E_0 is a constant field for breaking the symmetry of the double well, and E and ω are, respectively, the amplitude and the frequency of the driving laser field. Hereafter we use units where $\hbar$ is equal to 1.

The Hamiltonian (1) can be easily reduced to a form which is convenient for the presentation of optical properties by employing the following unitary transformation:

$$U = \exp(\mathrm{i}\pi \sigma_y / 4) . \tag{3}$$

The Hamiltonian (1) then takes the form

$$H(t) = U H U^{-1} = -V(t) \sigma_x + \Delta \sigma_z . \tag{4}$$

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The time-dependent dipole moment is defined by

$$\mu(t) = \langle \psi | \sigma_x(t) | \psi \rangle. \tag{5}$$

The dynamics of $\mu(\tau)$ is governed by the integro-differential equation [8]

$$\mathrm{d}\mu(\tau)/\mathrm{d}\tau = -(\epsilon)^2 \int_0^\tau \mathrm{d}\tau_1 \ \cos[a\sin(\tau) - a\sin(\tau_1) + b(\tau - \tau_1)]\mu(\tau_1) \tag{6}$$

with the initial condition $\mu(0) = 1$, and here we have made use of the following substitutions:

$$\tau = \omega t \qquad \mu(\tau) \longrightarrow \mu(\tau)/\mu(0)$$

as well the as the following definitions:

$$a \equiv 2\mu E/\omega \qquad b \equiv 2\mu E_0/\omega \qquad \epsilon \equiv \Delta/\omega.$$

To simplify equation (6), we shall transform this equation into a convolution-type integro-differential one in the case of a high-frequency driving field. Meanwhile, we put the reduced DC field intensity b into integer form (i.e. $b = 2\mu E_0/\omega = N$), since we have found that when one ratio of the reduced DC field strength to the photon energy becomes an integer, i.e., $2\mu E_0/\omega = N$ ($N = 0, 1, 2, \ldots$), the exact quasienergy crossing (and the localization of the electron) will occur if the other ratio of the reduced AC field strength to the photon energy, $2\mu E/\omega$, is a root of the ordinary Bessel function of Nth order simultaneously [9] (i.e. the localized condition can be simply expressed as $J_N(2\mu E/\omega) = 0$ which yields the fan structure in the parameter space). To do this, in the case of a high-frequency driving field (i.e. $\epsilon \ll 1$), we check all orders of the expansion with respect to ϵ and find that equation (6) can be rewritten as

$$\mathrm{d}\mu(\tau)/\mathrm{d}\tau = -(\epsilon)^2\,\mathrm{Re}\int_0^\tau \mathrm{d}\tau_1 \ J_0\{2a\sin[(\tau - \tau_1)/2]\}\exp[\mathrm{i}N(\tau - \tau_1)]\mu(\tau_1). \tag{7}$$

Now the kinetic equation (7) is of convolution type, and can be solved by using the Laplace transformation defined as

$$\mu(\lambda) = \int_0^\infty \mathrm{d}t \ \mathrm{e}^{-\lambda t}\mu(t). \tag{8}$$

The formal solution of equation (7) is

$$\mu(\lambda) = \frac{1}{\lambda + \Delta^2 K(\lambda)} \tag{9}$$

where

$$K(\lambda) = \int_0^\infty \mathrm{d}t \ \mathrm{e}^{-\lambda t} J_0[2a\sin(\omega t/2)]\cos[N\omega t]. \tag{10}$$

To find the integral (10) we expand the zeroth Bessel function into a Fourier series by the use of the following identity [10]:

$$J_0[2a\sin(\omega t/2)] = J_0{}^2(a) + 2\sum_{m=1}^{\infty} J_m{}^2(a)\cos(m\omega t). \tag{11}$$

Substituting equation (11) into equation (10), we obtain

$$K(\lambda) = \frac{\lambda J_0{}^2(a)}{\lambda^2 + (N\omega)^2} + \frac{J_N{}^2(a)}{\lambda} + \sum_{m=1}^{\infty} \frac{\lambda J_m{}^2(a)}{\lambda^2 + [(m+N)\omega)]^2} + \sum_{m\neq N}^{\infty} \frac{\lambda J_m{}^2(a)}{\lambda^2 + [(m-N)\omega)]^2}. \tag{12}$$

The long-time property of the solution of equation (7) is determined by the behaviour of $K(\lambda)$ at small λ. Therefore, we focus on the situation of λ being small at first. From equation (12), we can see that in this case the behaviour of $K(\lambda)$ is dominated by the second term when $J_N(a) \neq 0$. Keeping this in mind and using the inverse Laplace transform, we get

$$\mu(\tau) = \cos(\Omega_N \tau) = \mu_0(\tau) \tag{13}$$

where

$$\Omega_N = \epsilon J_N(a). \tag{14}$$

In order to obtain the high-frequency part of the spectrum, we substitute the solution (13) into equation (6), and perform the first iteration with respect to $\mu_0(\tau)$, getting

$$\mu(\tau) = \cos(\Omega_N \tau) - \epsilon \sum_{k=1}^{\infty} (-1)^k C_{N,k} \{\cos[(k - \Omega_N)\tau] - \cos[(k + \Omega_N)\tau]\} \tag{15}$$

where

$$C_{N,k} = [J_{N+k}(a) + J_{N-k}(a)]/2k. \tag{16}$$

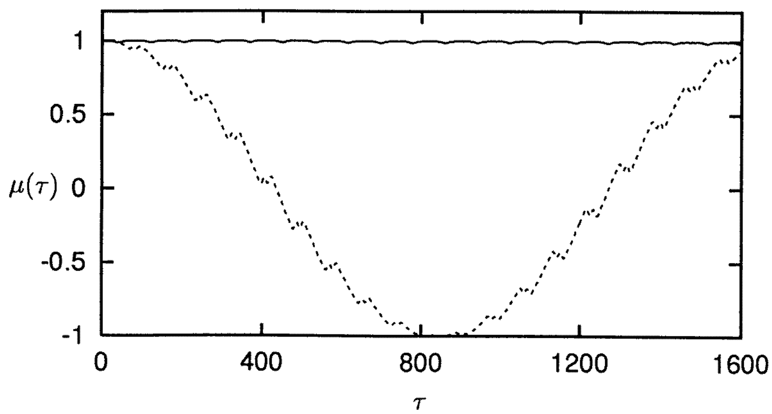

Figure 1. The induced dipole $\mu(\tau)$ as a function of the scaled time τ. The solid line shows $\mu(\tau)$ for $a = 2\mu E/\omega = 5.14$, $b = 2\mu E_0/\omega = 2.00$ (the case where $\Omega_N = 0$), while the dotted line shows $\mu(\tau)$ for $a = 2\mu E/\omega = 3.20$, $b = 2\mu E_0/\omega = 2.00$ (the case where $\Omega_N \neq 0$). Here $\epsilon = 0.05$.

From equation (15) we can see that the Fourier transform $\mu(\Omega)$ has a peak at the frequency Ω_N (in units of ω) which stemmed from the first term on the right-hand side of equation (15). If Ω_N is small, this corresponds to the low-frequency generation (LFG). The other terms on the right-hand side of equation (15) represent the high-frequency parts of the spectrum. Note that the intensity of the LFG peak is very high, as compared to that of the other peaks, since the transition dipole $\mu(\tau)$ is dominated by the first term, $\cos(\Omega_N \tau)$, under the approximation $\epsilon \ll 1$. In the extreme low-frequency limit, $\Omega_N \to 0$, the transit dipole $\mu(\tau)$ will approach unity, $\mu(\tau) \to 1$, indicating localization. Otherwise $\mu(\tau)$ will oscillate between 1 and -1. This feature is confirmed by our numerics depicted in figure 1, where the dipole $\mu(\tau)$ is plotted as a function of the scaled time τ. To generate the graph, we have taken $\epsilon = 0.05$. The solid line shows an example where we let the parameters $a = 5.14$ and $b = 2$ so as to ensure that the localization condition is met, i.e., $\Omega_N = 0$. It is clearly seen in this case that $\mu(\tau)$ is almost equal to a constant which is close to one at all times, plus a term oscillating with small amplitude and high frequency. This coincides with the findings of [9]. The dotted line illustrates the situation with $a = 3.2$ and $b = 2$ that results in the value of Ω_N given by equation (14) being finitely small. The time evolution of $\mu(\tau)$ shows a large-amplitude–low-frequency component and a high-frequency–low-amplitude behaviour, as predicted by the theory (equations (15) and (16)).

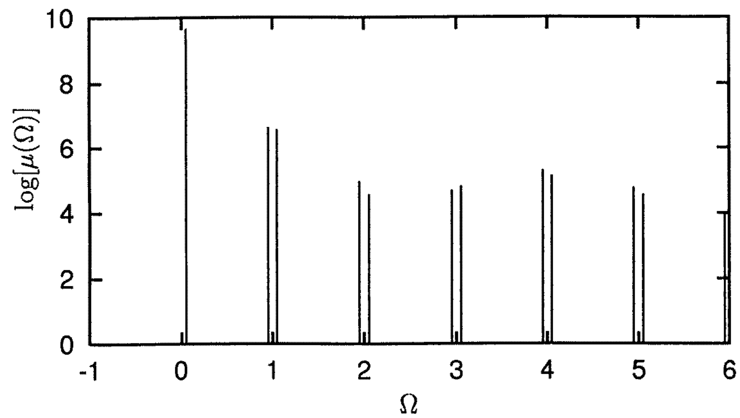

Figure 2. The numerically calculated emission spectrum with $a = 2\mu E/\omega = 3.20$, $b = 2\mu E_0/\omega = 2.00$, and $\epsilon = 0.05$.

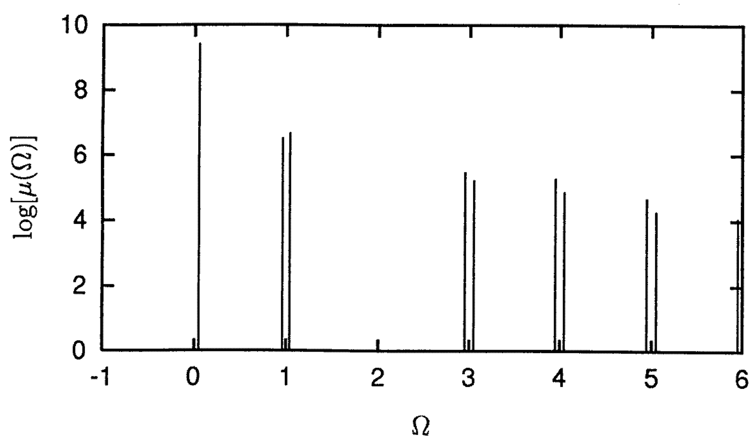

Figure 3. The numerically calculated emission spectrum with $a = 2\mu E/\omega = 3.055$, $b = 2\mu E_0/\omega = 1.00$, and $\epsilon = 0.05$.

The equations (15) and (16) need more comment. At first, the spectrum $\mu(\Omega)$ consists of a number of doublets, $\Omega = k + \Omega_N$ and $\Omega = k - \Omega_N$; here k includes both even and odd harmonics. This is different from the pure AC-field driving case where the spectrum consists of doublets at even harmonics with vanishing amplitudes at odd ones [7]. Figure 2 shows an example for these doublets, $\Omega = k + \Omega_N$ and $\Omega = k - \Omega_N$, as well as the low-frequency component Ω_N. This plot is generated by the Fourier transformation of the time-dependent dipole moment numerically, by the use of equation (5). Note that we label the vertical axis with the absolute amplitudes of harmonic generations in $\mu(\Omega)$.

Secondly, from equations (15) and (16) we can see that some doublets will disappear when $C_{N,k} = 0$. This can be fulfilled by the appropriate choice of the parameters. For instance, when $a = 3.055$, $b = N = 1$, we have $J_3(3.055) + J_{-1}(3.055) = 0$. Therefore, $C_{1,2} = 0$. Correspondingly, the amplitude of the second-harmonic doublet should be eliminated. This becomes transparent in figure 3.

Finally, when $a = 2\mu E/\omega$ is taken to be a zero of the J_N Bessel function (which is the localization condition in [9]), the two satellites will merge and destructively interfere, yielding a vanishing spectrum except for a zero-frequency term. We have not plotted this here, but it is manifested in equations (15) and (16).

In conclusion, we have studied the emission spectrum of an electron in a two-level system driven by DC–AC fields. We have shown that, in general, the emission spectrum consists of a static component, at low frequency, Ω_N(LFG), and doublets at frequencies $k + \Omega_N$ and $k - \Omega_N$ for $k = 1, 2, 3, \ldots$. The signature of the fan structure of [9] in the emission spectrum is transparent: it yields dominantly the zero-frequency component. The phenomenon of low-frequency generation is observed, and the conditions under which the LFG is intense and the localization is strong are obtained. The amplitudes of all lines and Ω_N depend on the field parameters a and b. Therefore, by making a proper choice of the field parameters, we can selectively eliminate any one of the doublets in the spectrum.

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